Technical Notes

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A Source-Equality Kutta Condition for Panel Methods

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Introduction

EARLIER surface singularity methods developed to solve lifting potential flow problems used a combination of external source panels and internal doublet distributions of a predetermined shape to provide the displacement effect and the circulation, respectively. Provided that the boundary conditions lead to a well-posed problem, i.e., the solution is unique, any arbitrary combination of sources and doublets should give the same external flowfield. Experience in applying these methods, however, has shown the numerical results to be rather sensitive to the shape assumed for the doublet distribution and to the particular implementation of the Kutta condition used to determine the total circulation.

When the prescribed doublet distribution does not represent the actual chordwise loading, the source distribution provides a doublet effect by assuming large values of opposite sign on the upper and lower surfaces of the wing.[1] Large variations in the source strength can induce leakage effects at the panel edges that adversely affect the solution in certain configurations (e.g., analysis of internal flow). These adverse effects can be minimized, either by using appropriate doublet distributions or by adopting an optimizing procedure such as that of Ref. 2.

A major problem in these methods is, however, the practical difficulty in applying the Kutta trailing-edge condition. Either the flow-tangency or the pressure-equality conditions currently used in panel methods to approximate the physical Kutta condition of regular velocity must be applied at a small distance away from the trailing edge. Therefore, significant variations in the overall circulation and local flow properties can result from different choices for the type and/or the location of the application point of the numerical Kutta condition.[3]

The purpose of this Note is to propose a source-equality condition that can be applied at the trailing edge. Its application to low- and higher-order panel methods is presented to quantify the improvements achieved using the present model vs two conventional forms.

Basis of the Method

In the modeling of thick wings, the trailing edge is the common edge of an arbitrary number of source and doublet panels (Fig. 1). Let A be a point a small distance η away from this edge, t a vector through A perpendicular to the edge, and n a vector perpendicular to both t and the trailing edge. According to Ref. 4, the velocity V_A at the point A can be expressed as

$$V_A \cdot n = \frac{1}{4\pi}\left[\frac{2\Delta\mu}{\eta} + 2\Delta\mu' \ln(|\eta|) + \text{regular terms}\right] \tag{1a}$$

$$V_A \cdot t = \frac{1}{4\pi}\left[-2\Delta\sigma \ln(|\eta|) + \text{regular terms}\right] \tag{1b}$$

where $\Delta\mu$ and $\Delta\sigma$ are the jumps in the doublet and source strengths across the edge and $\Delta\mu'$ the jump in the derivative of the doublet strength in the direction perpendicular to the edge. As point A approaches the trailing edge, i.e., $\eta \to 0$, all three terms shown in Eqs. (1) become singular.

In a rather broad sense, the Kutta condition postulates that the velocity must be regular at the trailing edge. For panel methods, this strictly implies that at the trailing edge all of the singularity jumps must vanish, i.e.,

$$\Delta\mu_{TE} = 0, \qquad \Delta\mu'_{TE} = 0, \qquad \Delta\sigma_{TE} = 0 \tag{2}$$

Among the currently used singularity schemes, only the symmetrical singularity method of Ref. 1 explicitly fulfills all three conditions of Eq. (2). All the others satisfy explicitly only the doublet conditions, the source condition being satisfied within the numerical accuracy by the solution.

For methods using doublet distributions of a given shape, which must satisfy $\Delta\mu_{TE} = 0$ and $\Delta\mu'_{TE} = 0$, Eqs. (2) suggest an alternative way to determine the source and doublet combination by equating the source strengths at the trailing edge.

Although this possibility has been mentioned by Hess,[3] it has not been used in numerical applications. Indeed, this source-equality condition offers two practical advantages over the flow-tangency or the pressure-equality conditions so far used: 1) it is enforced exactly at the trailing edge, thus avoiding any influence from the distance between the application point and the trailing edge; and 2) it prescribes a condition for the source strength, thus eliminating any numerical inaccuracy related to the velocity evaluation in a region where the flow properties become singular.

Applications

Higher-Order Panel Method

The present Kutta model was first applied in a higher-order panel method.[5] Following the idea of logically independent networks,[4] lifting surfaces are split into a convenient number of segments, each containing two source networks located on the upper and the lower surfaces; a doublet network on the camber surface and a doublet network to represent the wake. The singularity panels that form the network can be flat or curved, with linearly varying source and quadratically varying doublet distributions.

In the standard solution procedure, the Kutta condition is applied in a conventional form by enforcing the continuity of the doublet strength and its derivative between the inner doublet network and the wake network. In the new model, the doublet networks have predetermined chordwise distributions of the form

$$\mu(\xi) = (2-\xi^\nu)\xi^\nu \quad \text{in the interior network}$$
$$= 1 \quad \text{in the wake}$$

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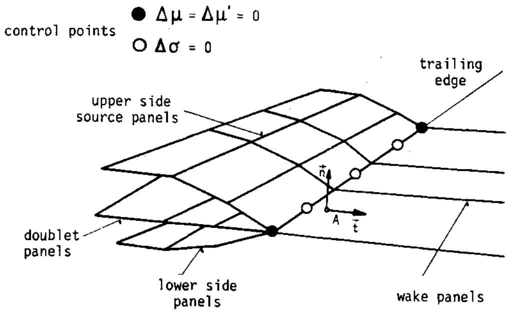

Fig. 1 **Panelling at the trailing edge.**

where $\xi = (x - x_{TE})/(x_{LE} - x_{TE})$ is a dimensionless chordwise coordinate. Clearly, these expressions satisfy the doublet conditions of Eqs. (2) for arbitrary ν. In order to determine the spanwise distribution of the circulation, the boundary conditions associated with the trailing-edge points have been modified (Fig. 1). These conditions are now of two types:

1) Equality of the local strength of the upper and lower side source panels enforced at the intermediate control points.

2) Continuity of the doublet strength and doublet derivative at the network edge control points.

The method was applied to the 2% thick straked wing configuration presented as a test case for the subsonic panel methods outlined in Ref. 6. Comparison of the results from the present method using both types of the solution procedure and the two datum solutions of Ref. 6 are presented in Fig. 2. Due to the strong three-dimensional character of the flow at the leading edge, significant variations are observed just inside the kink in the pressure distribution and especially in the lateral velocity distribution (Fig. 2). In the trailing-edge region, however, all the methods achieve comparable accuracy, regardless of the different modeling or of the total number of associated unknowns (see the inset in Fig. 2). On the contrary, earlier panel methods using doublet distributions of predetermined shapes and Kutta conditions of conventional forms showed poorer agreement with the datum solutions, especially in the trailing-edge region.[6]

Low-Order Panel Method

The MBB-standard panel method[7] is a first-order method that uses planar constant strength source panels on the wing surface and an internal vortex lattice extending from the wing camber surface to the wake. In the original version, the user predetermines the variation of the internal vortex strengths (a distribution proportional to the wing thickness is usually used). The unknown scaling factor for each wing strip is then determined by a flow-tangency Kutta condition enforced at the centroid of an auxiliary panel located on the extension of the wing camber surface. To implement the present formulation, each original Kutta equation has been replaced by the equality of the strengths of the corresponding lower and upper source panels adjacent to the trailing edge. In addition, the strength of the line vortex at the trailing edge has been set to zero, thus satisfying the conditions $\Delta\mu = 0$, $\Delta\mu' = 0$ of the equivalent doublet distributions.

A plane, 15% thick, straight wing of aspect ratio $AR = 4$ was analyzed at Mach = 0 and α = 10-deg angle of attack. The panel layout consists of four equally spaced spanwise segments of eight chordwise panels each. Figure 3 presents the comparison of the results computed from the source-equality and flow-tangency conditions, the latter applied at different distances $\Delta x_A/c$ from the trailing edge. Clearly, the circulation computed by the flow-tangency condition Γ_{FT} tends asymptotically to a limit value that corresponds to the circulation determined by the source-equality method Γ_{SE}. However, computations did not allow convergence at very small distances of the application point ($\Delta x_A/c < 10^{-5}$) from

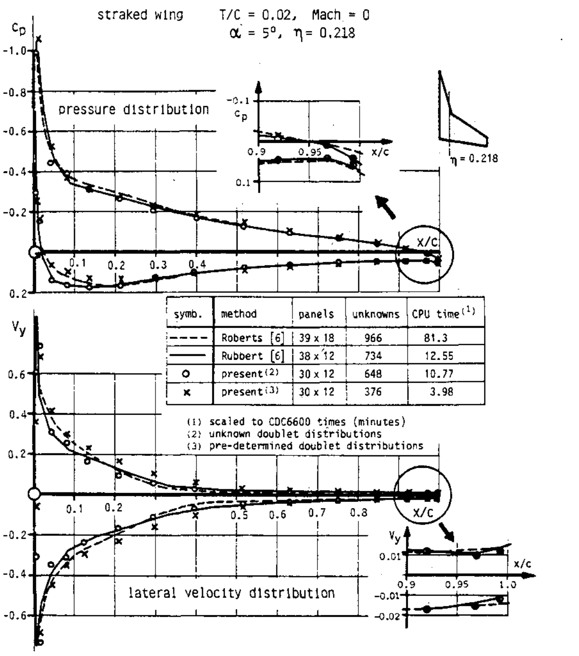

Fig. 2 **Comparison of higher-order method results.**

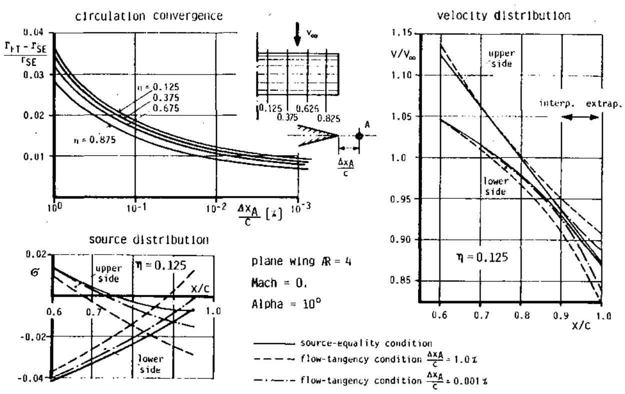

Fig. 3 **Comparison of low-order method results.**

the trailing edge, because of the development of round-off errors. As a result, equality of the source strength at the trailing edge cannot be obtained by the flow-tangency method. The extrapolation of the upper and lower side velocity distributions toward the trailing edge does not lead to the same value, as is the case when source-equality is directly enforced (Fig. 3).

Conclusion

Examination of the singular terms in the velocity induced by source and doublet distributions at panel edges has allowed the derivation of a source-equality trailing-edge condition. By this condition, the exact Kutta condition of regular velocity is applied exactly at the trailing edge. This eliminates the error introduced by enforcing the Kutta condition at a small but finite distance from the trailing edge, which is necessary when using the flow-tangency or pressure-equality conditions. Application of the present model to higher- and lower-order panel methods with doublet distributions of given shapes showed significant advantages over the conventional forms of the Kutta condition currently used in surface singularity methods of this type.

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Further Solutions in Streamwise Corner Flow with Wall Suction

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Introduction

IN Ref. 1, Barclay and El-Gamal have given solutions for the laminar boundary-layer flow along a rectangular streamwise corner when the flow is subjected to a uniform suction at the walls. Here the boundary-layer problem is reconsidered for the somewhat simpler case where the suction is proportional to the square root of the reciprocal of the local Reynolds number R_x. Introducing $R_x \equiv 2U_x/\nu$ but otherwise adopting the same notation as in Ref. 1, we therefore replace the suction velocity $v_s(x) = \text{const}$ with the condition that $v_s(x) = -\epsilon U R_x^{-1/2}$, $\epsilon = \text{const}$. Noting that σ (Ref. 1) is now independent of x (in fact, $\sigma = \epsilon$) the boundary-layer equations and boundary conditions are obtained directly from Eqs. (2-7) in Ref. 1 to be

$$-(\eta u^* - v^*)\frac{\partial u^*}{\partial \eta} - (\zeta u^* - w^*)\frac{\partial u^*}{\partial \zeta} = \nabla^2 u^* \tag{1a}$$

$$-u^*\left(v^* + \eta\frac{\partial v^*}{\partial \eta} + \zeta\frac{\partial v^*}{\partial \zeta}\right) + v^*\frac{\partial v^*}{\partial \eta} + w^*\frac{\partial v^*}{\partial \zeta} = \nabla^2 v^* - \frac{\partial p^*}{\partial \eta} \tag{1b}$$

$$-u^*\left(w^* + \eta\frac{\partial w^*}{\partial \eta} + \zeta\frac{\partial w^*}{\partial \zeta}\right) + v^*\frac{\partial w^*}{\partial \eta} + w^*\frac{\partial w^*}{\partial \zeta} = \nabla^2 w^* - \frac{\partial p^*}{\partial \zeta} \tag{1c}$$

$$-\eta\frac{\partial u^*}{\partial \eta} - \zeta\frac{\partial u^*}{\partial \zeta} + \frac{\partial v^*}{\partial \eta} + \frac{\partial w^*}{\partial \zeta} = 0 \tag{1d}$$

$$\begin{aligned}
&\eta = 0; && u^* = 0, && v^* = \epsilon v_c^*(\zeta), && w^* = 0\\
&\zeta = 0; && u^* = 0, && v^* = 0, && w^* = \epsilon v_c^*(\eta)\\
&\eta \to \infty; && u^* = \bar{u}(\zeta), && v^* = \bar{w}(\zeta), && w^* = \bar{v}(\zeta)\\
&\zeta \to \infty; && u^* = \bar{u}(\eta), && v^* = \bar{v}(\eta), && w^* = \bar{w}(\eta)
\end{aligned} \tag{2}$$

where $\bar{u}(\eta), \bar{v}(\eta), \bar{w}(\eta)$ are the solutions of

$$-(\eta\bar{u} - \bar{v})\frac{\partial \bar{u}}{\partial \eta} = \frac{\partial^2 \bar{u}}{\partial \eta^2} \tag{3}$$

$$-(\eta\bar{u} - \bar{v})\frac{\partial \bar{w}}{\partial \eta} - \bar{u}\bar{w} = -\lim_{\eta\to\infty} \bar{v} + \frac{\partial^2 \bar{w}}{\partial \eta^2} \tag{4}$$

$$-\eta\frac{\partial \bar{u}}{\partial \eta} + \frac{\partial \bar{v}}{\partial \eta} = 0 \tag{5}$$

subject to

$$\bar{u}(0) = 0, \qquad \bar{u}(\infty) = 1, \qquad \bar{v}(0) = -\epsilon \tag{6a}$$

$$\bar{w}(0) = 0, \qquad \bar{w}(\infty) = \bar{v}(\infty) \tag{6b}$$

As before, $v_c^*(t)$ is considered to be indeterminate and a solution to Eq. (1) must involve an assumption for the form of $v_c^*(t)$ satisfying the conditions $v_c^*(0) = 0$ and $v_c^*(\infty) = -1$.

Solution at the Side Edge ($\zeta \to \infty$)

Equations (1) and (3-5) above are the same as for zero suction, but their solutions are parametrically dependent on the proportionality constant ϵ appearing in the boundary conditions. The zero suction case $\epsilon = 0$ has been solved by Rubin[2] and Rubin and Grossman.[3] Equations (3) and (5), with Eq. (6a), are uncoupled from Eqs. (4) and (6b) and have been solved for $\bar{u}$ and $\bar{v}$ by Schlichting and Bussmann.[4] Improved numerical results have been given by Emmons and Leigh.[5]

Letting $\bar{u} = F'(\eta)$, $\bar{w} = \beta H'(\eta)$ where $\beta = \lim_{\eta\to\infty} \bar{v}$, and substituting in Eqs. (4) and (6b) we obtain Rubin's equations

$$H''' + (H'F)' = 1, \qquad H'(0) = 0, \qquad H'(\infty) = 1$$

for which the solution is

$$H'(\eta) = F''(\eta)\int_0^\eta \frac{(t-\beta)}{F''(t)}\,dt \tag{7}$$

$\bar{w}(\eta)$ is shown in Fig. 1. The outflow β from the boundary layer is zero at $\epsilon = 0.731$ (approx.) and $\bar{w}$ is therefore zero everywhere at the side edge for $\epsilon = 0.731$. This interesting result separates the cross flow into two types, one displaying a region of flow reversal ($0 \le \epsilon < 0.731$) and one in which $\bar{w}(\eta)$ is everywhere negative ($\epsilon > 0.731$).

When $\epsilon \sim 4$ or larger, all of the velocity components have practically reached asymptotic forms valid for large ϵ. It has been shown by Watson[6] in his analysis for two-dimensional flows that the asymptotic solutions for $\bar{u}$ and $\bar{v}$ take the forms

$$\bar{u} = 1 - e^{-\epsilon\eta}, \qquad \bar{v} = -\epsilon \tag{8}$$

It may also be shown that the asymptotic solution for $\bar{w}$ is

$$\bar{w} = -\epsilon(1 - e^{-\epsilon\eta}) = -\epsilon\bar{u} \tag{9}$$

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